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Thermal-Stress effects on nonlinear thin film Waveguide Sensors

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We analyze the modelling of nonlinear waveguide sensors. The effect of thermal-stress on the optical performance of nonlinear symmetrical sensor is studied. The mathematical forms of the dispersion equation and thermal stress sensitivity are analytically derived and plotted numerically. It is found that the thermal sensitivity of the sensor can be controlled by tuning the core size, by changing loading materials, and by carefully selecting the materials.

Keywords: nonlinear, optical sensor, thermal effect, thermal sensitivity, thermal stress.

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Introduction

Integrated Optical sensor field is motivated by the expectation that optical sensors have significant advantages compared to conventional sensor types, i.e., their immunity to electromagnetic interference, the use in aggressive environment, capability of performing multi-functional sensor on one chip, and higher sensitivity [1,2]. The properties of waveguide sensing in dielectric films have been studied intensively for a number of years and have resulted in a large number of devices. A nonlinear optically surface waves that propagate along planar interfaces between dielectric media, has been investigated considerably because of their applications in all optical switches [3-5].

The essential part of the optical waveguide sensor is the ability to transform, in an efficient way, a chemical or biological reaction in a measurable signal [6]. Substantial efforts have been made to enhance the sensitivity of waveguide sensors. Huang and Yan [7] studied the effect of thermal-stress on the sensitivity of three linear layers symmetrically optical waveguide sensors. El-Khozondar, et. al. [8] showed that different stress states can control the stress sensitivity of three layers asymmetrical optical waveguide sensor which consists of dielectric core surrounded by linear substrate and nonlinear cladding.

In this work, three layers nonlinear symmetrical optical waveguide sensor is considered. The substrate and cladding are taken to be nonlinear media surrounding a dielectric core. In section 2, the thermal-stress effects on temperature sensitivity of optical performance has been introduced. The field equations and the dispersion equation are derived analytically in section 3. Section 4 is dedicated to evaluate the temperature sensitivity for nonlinear medium. Numerical results are discussed in section 5, followed by conclusion in section 6.

I. Thermal-Stress effects on temperature sensitivity of optical performance

The practical waveguides is usually under anisotropic and inhomogeneous stress state. The dielectric constant of anisotropic and inhomogeneous medium, ε, is

$$\varepsilon = \begin{bmatrix} n_{xx}^2 & n_{xy}^2 & n_{xz}^2 \\ n_{xy}^2 & n_{yy}^2 & n_{yz}^2 \\ n_{xz}^2 & n_{yz}^2 & n_{zz}^2 \end{bmatrix}, \quad (1)$$

with n_{xx}, n_{yy}, n_{zz}, n_{xy}, n_{xz}, n_{yz} are function of temperature, stress, and wavelength [9]. The relation between refractive index and temperature (thermo-optic relation) is

$$\frac{\partial n}{\partial T} = Bn, \quad (2)$$

where T is temperature and B is the thermo-optic coefficient, which is usually a function of refractive index, wavelength, and temperature. Combining the effect of stress and temperature on the dielectric film gives the refractive index change as

$$\Delta\begin{pmatrix} n_{xx} \\ n_{yy} \\ n_{zz} \\ n_{yz} \\ n_{xz} \\ n_{xy} \end{pmatrix} = B\Delta T\begin{pmatrix} n_{xx} \\ n_{yy} \\ n_{zz} \\ n_{yz} \\ n_{xz} \\ n_{xy} \end{pmatrix} - \begin{bmatrix} C_1 & C_2 & C_2 & 0 & 0 & 0 \\ C_2 & C_1 & C_2 & 0 & 0 & 0 \\ C_2 & C_2 & C_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & C_3 \end{bmatrix}\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix} \quad (3)$$

where C_1, C_2, and C_3 are stress-optics constants depend on Young's modulus, shear modulus and Poisson's ratio; $\Delta n = n(T) - n(T0)$, $\Delta T = T - T_0$; σ_{xx}, σ_{yy}, σ_{zz}, σ_{yz}, σ_{xz}, and σxy are stress components.

As the waveguide is usually very long in one direction, denoted as z, the shear stresses in this direction can be ignored. Therefore the dielectric tensor becomes:

$$\varepsilon = \begin{bmatrix} n_{xx}^2 & n_{xy}^2 & 0 \\ n_{xy}^2 & n_{yy}^2 & 0 \\ 0 & 0 & n_{zz}^2 \end{bmatrix}, \quad (4)$$

and the stress state in the core can be expressed as

$$\sigma_{xx} = g_x E\Delta\alpha\left(T - T_0\right) + \sigma_{rx}, \quad (5)$$

$$\sigma_{yy} = g_y E\Delta\alpha\left(T - T_0\right) + \sigma_{ry}, \quad (6)$$

$$\sigma_{zz} = g_z E\Delta\alpha\left(T - T_0\right) + \sigma_{rz}, \quad (7)$$

$$\sigma_{yz} = \sigma_{xz} = \sigma_{xy} = 0, \quad (8)$$

where E is Young's modulus of the core; $\Delta\alpha = \alpha_{\text{cladding}} - \alpha_{\text{core}}$ is the thermal-expansion coefficient mismatch between cladding and the core; σ_{rx}, σ_{ry}, and σ_{rz} are residual stresses along x, y, z; g_x, g_y, and g_z, are loading parameters which in most cases need to be determine numerically. For simplicity, thermal stress is assumed to be due to thermal mismatch between core and cladding. The loading parameters in this case are: $g_x = 0$ and $g_y = g_z = 1/(1-\nu)$, where ν is Poisson's ratio of the core [10]. To study the stress effect on nonlinear waveguides we assume that the core to be under hydrostatic stress state, i.e., $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma$.

II. The electric field in the symmetrical nonlinear optical sensor

The waveguide sensor is assumed to consist of anisotropic inhomogeneous core as described in section 1 surrounded by nonlinear substrate and cladding. We assume a core of finite linear layer occupies the region $0 \le z \le t$. The two surrounding nonlinear layers occupy the regions $z < 0$ and $z > t$ as substrate and cladding respectively. The dielectric function of the nonlinear layers is chosen to be Kerr like function which is characterized by,

$$\varepsilon_i = \varepsilon_{oi} + \alpha_i E^2 \quad (9)$$

where ε_{oi} is a frequency independent linear part, α_i is the nonlinearity coefficient, and i =c, s stands to cladding or substrate [11-13]. Only TE modes (S-polarized waves) are considered which propagate along the x-axis and has the oscillating form expressed as follow,

$$\vec{E} = (0, E_y, 0)\, e^{\left(ik_0(n_e x - ct)\right)},$$

and $$\vec{H} = (H_x, 0, H_z)\, e^{\left(ik_0(n_e x - ct)\right)}, \quad (10)$$

where $n_e = k/k_0$ is the effective index, k_0, c are the wave number and the speed of light in free space, respectively. For TE modes, applying equation 2 into Maxwell's equations for fields in free space results in the wave equations for cladding and substrate [11-13] and for core [10] as follows,

$$\frac{\partial^2 E_y}{\partial z^2} - k_0^2\left(n_e^2 - \varepsilon_c\right)E_y + \alpha_c k_0^2\left(E_y^3\right) = 0 \quad \text{(cladding)} \quad (11)$$

$$\frac{\partial^2 E_y}{\partial z^2} + k_0^2\left(n^2 - {n_e}^2\right)E_y = 0 \quad \text{(core)} \quad (12)$$

$$\frac{\partial^2 E_y}{\partial z^2} - k_0^2\left({n_e}^2 - \varepsilon_s\right)E_y + \alpha_s k_0^2\left(E_y^3\right) = 0 \quad \text{(substrate)} \quad (13)$$

For $\alpha > 0$ the solutions of equations 11 and 13 respectively are given by [10-12],

$$E_c = \sqrt{\frac{2}{\alpha_c}}\frac{A_c q_c}{\cosh\left(k_0 q_c (z - z_0)\right)}, \quad (14)$$

$$E_s = \sqrt{\frac{2}{\alpha_s}}\frac{A_s q_s}{\cosh\left(k_0 q_s (z - z_0)\right)}, \quad (15)$$

where z_0 is a constant related to the power of the waveguide (at $z = z_0$ the power is maximum), $q_i = \sqrt{n_e^2 - \varepsilon_i}$, i = c, s which denotes the cladding and substrate respectively. For $n^2 > n_e^2$ the solution of equation 12 can be expressed as,

$$E_f = A_f \cosh(k_O\, p\, z) + B_f \sinh(k_O\, p\, z), \quad (16)$$

where $p = \sqrt{n^2 - n_e^2}$.

Applying boundary conditions which states that E_y and H_z are continuous across the boundary, z = 0 and z = t results in four homogenous linear equations. The electric field at the core is

$$E_f = A_f[\cosh(k_0 pz) + \frac{q_s}{p}\tanh(k_0 q_s z_0)\sinh(k_0 pz)], \quad (17)$$

and the dispersion equation is

$$k_0 tp - 2\tanh^{-1}(\frac{q}{p}\tanh C) - m\pi = 0, \quad (18)$$

where $q = q_c = q_s$, $\tanh C = \tanh(k_0 q_{z0})$

III. Temperature sensitivity for the nonlinear medium

Temperature sensitivity is defined as the rate of change of the effective refractive index with respect to temperature. To evaluate the temperature sensitivity, the dispersion equation (equation 18) is differentiated with respect to temperature T, and the quantities p and q are considered as a function of T i.e. $p = \sqrt{n^2(T) - n_e^2(T)}$ and $q = q_i = \sqrt{n_e^2(T) - n_1^2(T)}$, where $n_1^2 = \varepsilon_i$. Differentiating p and q yields

$$dp = \frac{n\, dn - n_e dn_e}{p} \quad \text{and} \quad dq = \frac{n_e\, dn_e - n_1 dn_1}{q}.$$

Then, the dispersion equation (equation 18) is differentiated with respect to temperature T as follows

$$k_0 t dp - 2\left(\frac{\tanh(k_0 z_0 q)\, dq / p \ - q \tanh(k_0 z_0 q)\, dp / p^2 + (1 - \tanh^2(k_0 z_0 q)\, q\, k_0 z_0 dq) / p}{1 - \tanh^2\left(\tanh^{-1}(q \ \tanh(k_0 z_0 q) / p)\right)} \right) = 0 \qquad (19)$$

Substituting dp and dq , and use $dn_e = S_T$, equation 19 is simplified to

$$S_T = [F\, n\, dn\, q + G\, n_1 dn_1 p] / [n_e (F\, q + G\, p)] \qquad (20)$$

$$F = 1/2\ k_0 t (p^2 + q^2 \tanh^2 C) + q \tanh C \qquad (21)$$

$$G = \tanh C\ p + q(1 - \tanh^2 C) k_0 z_0 p \qquad (22)$$

IV. Numerical results

The analytical equations derived in the previous sections can be used to describe the planer waveguide sensors. To solve these equations, the following values of the parameters are used in the numerical calculations: $n_1 = 2$, $\lambda = 0.83$ µm, $dn_1 = 1 \cdot 10^{-5}$ 1/°C, $dn = 2 \cdot 10^{-5}$ 1/°C, $n_0 = 3.5$. The core thickness (t) has the values in the range 0 to 1.5 µm, also the quantity tanh C varies in the interval (-1 , 1). First, the effective refractive index is computed by solving the dispersion equation 18. Then the obtained effective refractive index is substituted into equation 20 to compute thermal sensitivity.

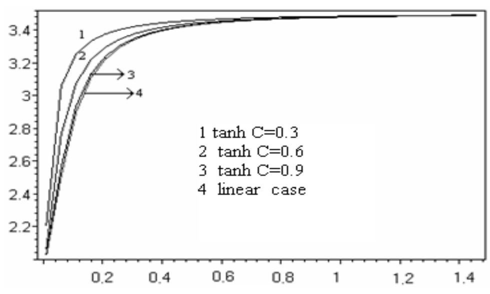

Fig. 1. the effective index (ne) as a function of core thickness (t) for m = 0.

Figure 1 shows the relation between the effective refractive index (n_e) for the zero mode (m = 0) and core thickness (t). The curves 1, 2, and 3 refers to the nonlinear case with tanh C = 0.3, 0.6, 0.9 while curve 4 indicates the linear case where tanh C = 1. The effective index varies with core thickness. It has an inverse relation with tanh C and reachs the minimum values when tanh C = 1. The relation between the effective refractive index (ne) for different modes (m = 0, 1, 2) is plotted as a function of core thickness (t) for tanh C = 0.9 in Figure 2. The effective index varies for different modes vary with core thickness. The core thickness can control the number of modes. Figure 3 shows the temperature sensitivity for m = 0 as a function of the core thickness (t) for tanh C has the values, tanh C = 0.55, 0.7, 0.95. The temperature sensitivity increases as the core thickness increases for t < 0.2 µm and reaches a maximum value at which saturation takes place. This means that there is a considerable tolerance choosing the core thickness which has a significant importance in manufacturing. There is also a direct proportionality between temperature sensitivity and tanh C where the

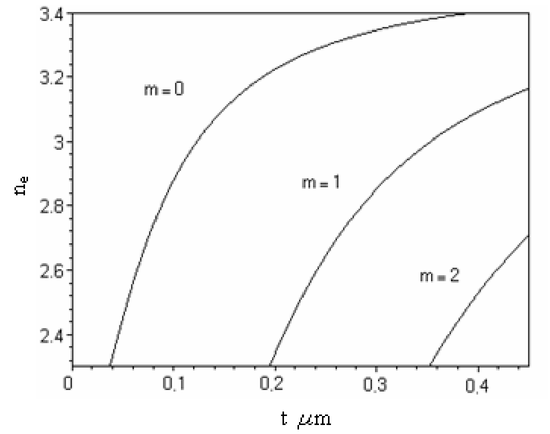

Fig. 2. Efrctive refractive index (ne) as a function of core thickness (t), for m = 0, 1, 2, and tanh C = 0.9.

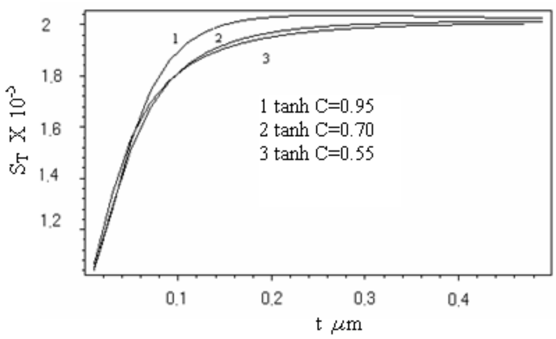

Fig. 3. Temperature sensitivity (ST) as a function of core thickness (t) for m = at different values of tanh C.

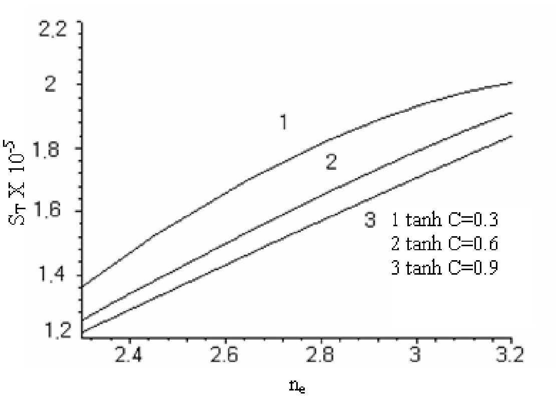

Fig. 4. The temperature sensitivity (ST) versus the effective refractive index (ne) for m = 0 at different values of tanh C.

value of the maximum sensitivity increases as the value of tanh C increases.

The temperature sensitivity (ST) versus the effective refractive index (ne) for m = 0 is shown in figure 4 where tanh C assumes the following values: 0.3, 0.6, 0.9. There is direct proportionality between ST and ne where the maximum value of ST occurs at tanh C = 0.3. ST is drawn as function of ne for m = 0, 1, 2 in figure 5 at tanh (C) = 0.7. The effective refractive index (ne) increases as the temperature sensitivity (ST) increases. The maximum value of ST occurs at m = 2.

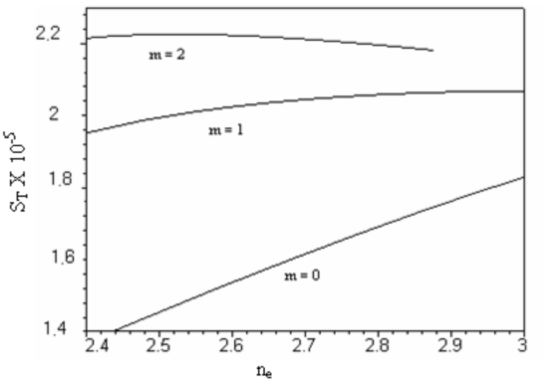

Figure 5: Temperature sensitivity (ST)as a function of effective refractive index (ne) for different modes at tanh C = 0.7.

Fig. 5. Temperature sensitivity (ST) as a function of effective refractive index (ne) different modes at tanh C = 0.7.

Conclusion

A symmetrical nonlinear optical waveguide sensor behavior have been studied subjected to thermal-stress and hydrostatic stress. It is found that the temperature sensitivity is directly proportional to the core thickness and the effective refractive index for different values of nonlinearity. As a result, the thermal stress sensitivity of the sensor can be controlled by the core size and the values of the stresses. Thermal stress in the waveguide can be controlled by carefully selecting the materials and loading methods, such as attaching a bimetal plate; therefore, the temperature sensitivity of the effective index can be carefully measured. Comparison between linear and nonlinear sensors assures that nonlinear sensors exhibit higher values of sensitivity. Moreover, the maximum sensitivity for nonlinear sensors appears at smaller core thickness than for linear sensor. This makes nonlinear sensors suitable for scientific and accurate sensing while linear configurations fit for commercial applications. Depending on tolerance in choosing waveguide width, it is possible to manufacture cheap sensors with suitable sensitivity.

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Ефект термозбудження в нелінійних тонкоплівкових хвилепровідних сенсорах

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Проаналізовано модель нелінійного хвилепровідного сенсора. Вивчено ефект термічнгого збудження оптичних властивостей в нелінійних симетричних сенсорах. Представлено математичну дисперсію формул та чутливість до термозбудження в аналітичному та графічному вигляді. Виявлено що, термочутливість сенсорів може контролюватись за допомогою зміни розмірів, процентного набору матеріалів та вибором самого матеріалу сенсора.

Ключові слова: нелінійний, оптичний сенсор, термічний ефект, термічна чутливість, термічне збудження.